Golden Ratio, Fibonacci Numbers, and the Golden Spiral

by

Troy Henderson

University of Mobile
The heavens are telling of the glory of God; and their expanse is declaring the work of His hands. Day to day pours forth speech, and night to night reveals knowledge. There is no speech, nor are there words; Their voice is not heard. Their line has gone out through all the earth, and their utterances to the end of the world.

Psalm 19:1-4a (NAS)
Golden Rectangle

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Golden Rectangle

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\[ x - 1 \]

1

1

x

X
Golden Rectangle

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\[
\frac{x}{1} = \frac{1}{x - 1}
\]
Golden Ratio

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\[
\frac{x}{1} = \frac{1}{x - 1}
\]

\[
x = 1 + \sqrt{5}, \quad x = 1 - \sqrt{5}
\]

\[
\phi = \frac{1}{1} = \frac{1}{\phi - 1} \approx 1.618
\]
Golden Ratio, Fibonacci Numbers, and the Golden Spiral

— Troy Henderson

\[
x \cdot (x - 1) = 1 \\
x^2 - x = 1 \\
x^2 - x - 1 = 0
\]

\[
x = \frac{1 + \sqrt{5}}{2}, \quad \frac{1 - \sqrt{5}}{2}
\]

\[
\phi = \frac{x}{1} = x = \frac{1 + \sqrt{5}}{2} \approx 1.618
\]
Mona Lisa

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Mona Lisa

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Parthenon

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Great Pyramid of Giza

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\[ a = 378' \]

\[ b = 481' \]
Great Pyramid of Giza

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\[
\frac{c}{a} = \frac{611.75}{378} \approx 1.618
\]

\[c \approx 611.75’\]

\[a = 378’\]

\[b = 481’\]
Human Arm

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6 7/8”  11 1/8”
Human Arm

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\[
\frac{11\frac{1}{8}}{6\frac{7}{8}} = \frac{89}{55} \approx 1.618
\]
Earth/Moon Radii

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Radius of Earth: \( a = 3,959 \) miles

Radius of Earth + Moon: \( b = 5,039 \) miles

\[ a^2 + b^2 = c^2 \]

\[ c = 6,408 \] miles

\[ \frac{c}{a} = \frac{6,408}{3,959} \approx 1.618 \]
Radius of Earth

\[ a = 3,959 \text{ miles} \]
Radius of Earth
\[ a = 3,959 \text{ miles} \]

Radius of Earth + Moon
\[ b = 5,039 \text{ miles} \]
Radius of Earth
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\[ a^2 + b^2 = c^2 \]
\[ c = 6,408 \text{ miles} \]
Radius of Earth
\[ a = 3,959 \text{ miles} \]

Radius of Earth + Moon
\[ b = 5,039 \text{ miles} \]

\[ a^2 + b^2 = c^2 \]
\[ c = 6,408 \text{ miles} \]

\[ \frac{c}{a} = \frac{6,408}{3,959} \approx 1.618 \]
Relative Mean Orbital Distances

<table>
<thead>
<tr>
<th>Planet</th>
<th>Mean distance (million km)</th>
<th>Relative mean (Mercury=1)</th>
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<tbody>
<tr>
<td>Mercury</td>
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### Relative Mean Orbital Distances

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<td><strong>1,640.67</strong></td>
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Fibonacci Squares

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Fibonacci Sequence

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\[ F_n = \{1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, \ldots \} \]
Fibonacci Sequence

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\[ F_n = \{1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, \ldots \} \]

\[ \frac{F_{n+1}}{F_n} = \left\{ \frac{1}{1}, \frac{2}{1'}, \frac{3}{2'}, \frac{5}{3'}, \frac{8}{5'}, \frac{13}{8'}, \frac{21}{13'}, \frac{34}{21'}, \frac{55}{34'}, \frac{89}{55'}, \frac{144}{89'}, \frac{233}{144'}, \ldots \right\} \]
Fibonacci Sequence

\[ F_n = \{1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, \ldots \} \]

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\[ \lim_{{n \to \infty}} \frac{F_{n+1}}{F_n} = \Phi = \frac{1 + \sqrt{5}}{2} \approx 1.618 \]
Finger Bones
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Human Arm

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\[
\frac{11\frac{1}{8}}{6\frac{7}{8}} = \frac{89}{55}
\]
“Make for yourself an ark of gopher wood; you shall make the ark with rooms, and shall cover it inside and out with pitch. This is how you shall make it: the length of the ark three hundred cubits, its breadth fifty cubits, and its height thirty cubits.”

Genesis 6:14-15 (NAS)
“Make for yourself an ark of gopher wood; you shall make the ark with rooms, and shall cover it inside and out with pitch. This is how you shall make it: the length of the ark three hundred cubits, its breadth fifty cubits, and its height thirty cubits.”

Genesis 6:14-15 (NAS)
“They shall construct an ark of acacia wood two and a half cubits long, and one and a half cubits wide, and one and a half cubits high. You shall overlay it with pure gold, inside and out you shall overlay it, and you shall make a gold molding around it. You shall cast four gold rings for it and fasten them on its four feet, and two rings shall be on one side of it and two rings on the other side of it. You shall make poles of acacia wood and overlay them with gold. You shall put the poles into the rings on the sides of the ark, to carry the ark with them. The poles shall remain in the rings of the ark; they shall not be removed from it. You shall put into the ark the testimony which I shall give you.

Exodus 25:10-16 (NAS)
Ark of the Covenant
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Ark of the Covenant

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Ark of the Covenant

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\[ \frac{2 \frac{1}{2}}{1 \frac{1}{2}} = \frac{5}{3} \]
“And you shall make the altar of acacia wood, **five** cubits long and **five** cubits wide; the altar shall be square, and its height shall be **three** cubits. You shall make its horns on its four corners; its horns shall be of one piece with it, and you shall overlay it with bronze.”

Exodus 27:1-2 (NAS)
Flower Petals

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Calla Lily

1 Petal
Flower Petals

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Euphorbia
2 Petals
Flower Petals

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Iris

3 Petals
Flower Petals

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Columbine
5 Petals
Flower Petals

Bloodroot
8 Petals
Black-eyed Susan

13 Petals
Flower Petals

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African Daisy
21 Petals
Flower Petals

Golden Ratio, Fibonacci Numbers, and the Golden Spiral — Troy Henderson

Shasta Daisy
34 Petals
Flower Petals

Golden Ratio, Fibonacci Numbers, and the Golden Spiral — Troy Henderson

Michaelmas Daisy
55 Petals
Golden Spiral

Gold Ratio, Fibonacci Numbers, and the Golden Spiral — Troy Henderson
Golden Spiral

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Golden Ratio, Fibonacci Numbers, and the Golden Spiral — Troy Henderson
Golden Spiral
Nautilus Shell

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— Troy Henderson
34 Counterclockwise Spirals
Sunflower

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Sunflower

Golden Ratio, Fibonacci Numbers, and the Golden Spiral — Troy Henderson
21 Clockwise Spirals
Hurricane Isaac

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Andromeda Galaxy

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For since the creation of the world His invisible attributes, His eternal power and divine nature, have been clearly seen, being understood through what has been made, so that they are without excuse.

Romans 1:20
That's all Folks!