Golden Ratio, Fibonacci Numbers, and the Golden Spiral

by

Troy Henderson

University of Mobile
The heavens are telling of the glory of God; and their expanse is declaring the work of His hands. Day to day pours forth speech, and night to night reveals knowledge. There is no speech, nor are there words; Their voice is not heard. Their line has gone out through all the earth, and their utterances to the end of the world.

Psalm 19:1-4a (NAS)
Golden Rectangle

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The diagram shows a rectangle divided into two rectangles: one with the label \( x - 1 \) and another with the label 1. The larger rectangle has a side length labeled \( x \). The ratios of the sides in the smaller rectangle are in the golden ratio, \( \frac{x}{x-1} \).
Golden Rectangle

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\[
\frac{x}{x - 1} = \frac{1}{1}
\]

\[
x = \frac{1}{x - 1}
\]
Golden Ratio

Golden Ratio, Fibonacci Numbers, and the Golden Spiral  —  Troy Henderson

\[
\frac{x}{1} = \frac{1}{x - 1}
\]

\[\phi = \frac{1 + \sqrt{5}}{2} \approx 1.618\]
Golden Ratio

Golden Ratio, Fibonacci Numbers, and the Golden Spiral — Troy Henderson

\[
\frac{x}{1} = \frac{1}{x - 1}
\]

\[x(x - 1) = 1\]

\[x^2 - x = 1\]

\[x^2 - x - 1 = 0\]

\[x = \frac{1 + \sqrt{5}}{2}, \frac{1 - \sqrt{5}}{2}\]

\(\phi = \frac{x}{1} = x = \frac{1 + \sqrt{5}}{2} \approx 1.618\)
Mona Lisa

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Mona Lisa

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Parthenon

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Great Pyramid of Giza

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Great Pyramid of Giza

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\[ a = 378' \]
\[ b = 481' \]
Great Pyramid of Giza

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\[ a = 378' \]
\[ b = 481' \]
\[ c \approx 611.75' \]
Great Pyramid of Giza

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$$\frac{c}{a} = \frac{611.75}{378} \approx 1.618$$

$c \approx 611.75'$

$b = 481'$

$a = 378'$
Human Arm

Golden Ratio, Fibonacci Numbers, and the Golden Spiral — Troy Henderson
Human Arm

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\[
\frac{11\frac{1}{8}}{6\frac{7}{8}} = \frac{89}{55} \approx 1.618
\]
Earth/Moon Radii

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Radius of Earth \( a = 3,959 \) miles
Radius of Earth + Moon \( b = 5,039 \) miles

\[
a^2 + b^2 = c^2
\]

\( c = 6,408 \) miles

\( \frac{a}{b} \approx 1.618 \),

\( a \approx \frac{3,959}{6,408} \approx 0.618 \)

Diagram of Earth and Moon.
Radius of Earth

\[ a = 3,959 \text{ miles} \]
Radius of Earth
\[ a = 3,959 \text{ miles} \]

Radius of Earth + Moon
\[ b = 5,039 \text{ miles} \]
Radius of Earth
\[ a = 3,959 \text{ miles} \]

Radius of Earth + Moon
\[ b = 5,039 \text{ miles} \]

\[ a^2 + b^2 = c^2 \]
\[ c = 6,408 \text{ miles} \]
Radius of Earth
\[ a = 3,959 \text{ miles} \]

Radius of Earth + Moon
\[ b = 5,039 \text{ miles} \]

\[ a^2 + b^2 = c^2 \]
\[ c = 6,408 \text{ miles} \]

\[ \frac{c}{a} = \frac{6,408}{3,959} \approx 1.618 \]
### Relative Mean Orbital Distances

**Golden Ratio, Fibonacci Numbers, and the Golden Spiral**  —  Troy Henderson

<table>
<thead>
<tr>
<th>Planet</th>
<th>Mean distance (million km)</th>
<th>Relative mean (Mercury=1)</th>
</tr>
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<tbody>
<tr>
<td>Mercury</td>
<td>57.91</td>
<td>1.00000</td>
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## Relative Mean Orbital Distances

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<td><strong>1.61874</strong></td>
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Fibonacci Squares

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Fibonacci Squares

Golden Ratio, Fibonacci Numbers, and the Golden Spiral — Troy Henderson
Fibonacci Sequence

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\[ F_n = \{1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, \ldots \} \]
Fibonacci Sequence

Golden Ratio, Fibonacci Numbers, and the Golden Spiral — Troy Henderson

\[ F_n = \{1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, \ldots \} \]

\[ \frac{F_{n+1}}{F_n} = \left\{ \frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}, \frac{21}{13}, \frac{34}{21}, \frac{55}{34}, \frac{89}{55}, \frac{144}{89}, \frac{233}{144}, \ldots \right\} \]
Fibonacci Sequence

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\[ F_n = \{1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, \ldots \} \]

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\[ \lim_{n \to \infty} \frac{F_{n+1}}{F_n} = \phi = \frac{1 + \sqrt{5}}{2} \approx 1.618 \]
Finger Bones

Golden Ratio, Fibonacci Numbers, and the Golden Spiral — Troy Henderson
Human Arm

Golden Ratio, Fibonacci Numbers, and the Golden Spiral — Troy Henderson

\[
\frac{11\frac{1}{8}}{6\frac{7}{8}} = \frac{89}{55}
\]
“Make for yourself an ark of gopher wood; you shall make the ark with rooms, and shall cover it inside and out with pitch. This is how you shall make it: the length of the ark three hundred cubits, its breadth **fifty** cubits, and its height **thirty** cubits.”

Genesis 6:14-15 (NAS)
“Make for yourself an ark of gopher wood; you shall make the ark with rooms, and shall cover it inside and out with pitch. This is how you shall make it: the length of the ark three hundred cubits, its breadth **fifty** cubits, and its height **thirty** cubits.”

Genesis 6:14-15 (NAS)

\[
\frac{50}{30} = \frac{5}{3} \approx 1.66
\]
“They shall construct an ark of acacia wood **two and a half** cubits long, and **one and a half** cubits wide, and **one and a half** cubits high. You shall overlay it with pure gold, inside and out you shall overlay it, and you shall make a gold molding around it. You shall cast four gold rings for it and fasten them on its four feet, and two rings shall be on one side of it and two rings on the other side of it. You shall make poles of acacia wood and overlay them with gold. You shall put the poles into the rings on the sides of the ark, to carry the ark with them. The poles shall remain in the rings of the ark; they shall not be removed from it. You shall put into the ark the testimony which I shall give you.

Exodus 25:10-16 (NAS)
Ark of the Covenant

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Ark of the Covenant

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\[ \frac{2\frac{1}{2}}{1\frac{1}{2}} = \frac{5}{3} \]
“And you shall make the altar of acacia wood, five cubits long and five cubits wide; the altar shall be square, and its height shall be three cubits. You shall make its horns on its four corners; its horns shall be of one piece with it, and you shall overlay it with bronze.”

Exodus 27:1-2 (NAS)
Flower Petals

Euphorbia
2 Petals
Flower Petals

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Iris
3 Petals
Flower Petals

Bloodroot
8 Petals
Black-eyed Susan
13 Petals
Flower Petals

African Daisy
21 Petals
Flower Petals

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Shasta Daisy
34 Petals
Flower Petals

Golden Ratio, Fibonacci Numbers, and the Golden Spiral — Troy Henderson

Michaelmas Daisy

55 Petals
Golden Spiral

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Golden Spiral

Golden Ratio, Fibonacci Numbers, and the Golden Spiral — Troy Henderson
Nautilus Shell

Golden Ratio, Fibonacci Numbers, and the Golden Spiral — Troy Henderson
Bighorn Sheep

Golden Ratio, Fibonacci Numbers, and the Golden Spiral — Troy Henderson
Sunflower

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Golden Ratio, Fibonacci Numbers, and the Golden Spiral — Troy Henderson
Sunflower

Golden Ratio, Fibonacci Numbers, and the Golden Spiral — Troy Henderson
34 Counterclockwise Spirals
Sunflower

Golden Ratio, Fibonacci Numbers, and the Golden Spiral — Troy Henderson
Sunflower

Golden Ratio, Fibonacci Numbers, and the Golden Spiral  —  Troy Henderson
21 Clockwise Spirals
Human Ear

Golden Ratio, Fibonacci Numbers, and the Golden Spiral — Troy Henderson
Hurricane Isaac

Golden Ratio, Fibonacci Numbers, and the Golden Spiral — Troy Henderson
Andromeda Galaxy

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For since the creation of the world His invisible attributes, His eternal power and divine nature, have been clearly seen, being understood through what has been made, so that they are without excuse.

Romans 1:20
That's all Folks!